Potentially Useful Formulae and Constants:

Speed of light, c= 3 x 10^8 ms^{-1}

Charge of the Electron, e = 1.6 x 10^{-19} C

Planck's Constant, h = 6.6 x 10^{-34} J s

Boltzmann's constant, k = 1.38 x 10^{-23} J K^{-1}

(The symbols below have their usual meaning)

Energy of a photon , $E = h\nu$

Resonator g factors: $g_1 = 1 - L/R_1$, $g_2 = 1 - L/R_2$

ABCD matrices for a curved mirror, thin lens, propagation, and a spherical dielectric interface:

$$\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}, \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ \dfrac{n_2 - n_1}{n_2 R} & n_1/n_2 \end{bmatrix}.$$

Finesse of a resonator with no loss apart from that associated with the finite reflectivity of the two mirrors. The mirror reflectivity is expressed in terms of its amplitude reflectivity:

$$F = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

Planck's radiation law: $u = \dfrac{8\pi h\nu^3}{c^3\left(e^{h\nu/(kT)} - 1\right)}$

Ratio of population densities in two levels separated by ΔE in energy: $n_2/n_1 = e^{-\Delta E/(kT)}$

Ratio of Einstein's A and B coefficients for a transition from level x to level y: $\dfrac{A_{yz}}{B_{yx}} = \dfrac{8\pi h\nu_o^3}{c^3}$

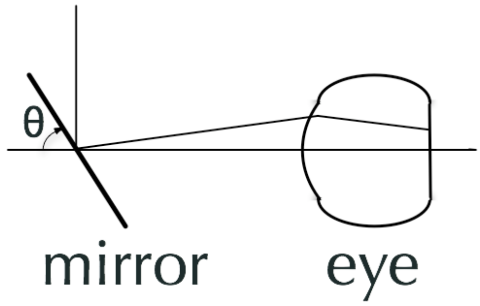
Q of a cavity resonance mode: $Q_c = \nu_0/\Delta\nu$ or $Q_c = 2\pi\nu_0$ [E / (dE/dt)] or $Q_c = 2\pi$ [E/E'] where E is the stored energy, dE/dt is the energy dissipated per second and E' is the energy dissipated per cycle.

Gain coefficient for a 4 level scheme in terms of population differences: $\kappa = \dfrac{c^2 g(\nu_r) A}{8\pi\nu_r^2}(n_3 - n_2)$

where c is the speed of light in the material and the rest of the symbols are interpreted as usual.

Population inversion at threshold for a 4 level laser scheme: $(n_3 - n_2)/N = W_{41}/(W_{41} + A_{32})$

1. You have been asked to design a device that can be used to perform laser surgery on the retina at the back of the eye. The design is based around steering a laser beam so that it falls exactly onto particular points on the retina using a rotating mirror. For simplicity the mirror has been designed to rotate about a point on its front surface, and this rotation point has also been placed on the optical axis of the eye system. The laser beam is aligned to hit the rotating mirror exactly upon this rotation point. The design for one axis of the steering system is shown in the figure below. This mirror redirects the laser into the eye, which for simplicity we will model as a curved dielectric surface followed by transparent media of refractive index n. The retina will be modelled as a flat surface at the back of the eye, which is a fixed distance behind the front surface (in other words we have chosen to ignore the slight lateral variation in length of the eye that arises because of the curvature of the surfaces).



Assume that the laser beam is perfectly collimated when it hits the mirror and also that it is travelling orthogonal to the optical axis before hitting the mirror. Assume that the refractive index of air is 1.

(i) Write down a sequence of ray matrices that describe the path of a ray that is just leaving the rotating mirror and which ends on a point on the retina *[8 marks]*

(ii) Simplify the ray matrix sequence found in part (i) into a single matrix *[4 marks]*

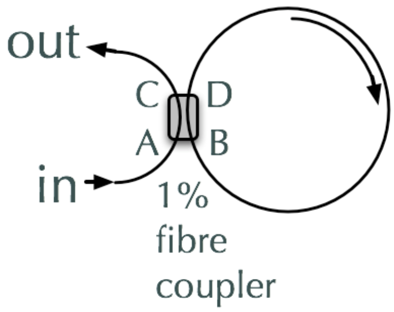
(iii) Calculate a ray vector, $\begin{pmatrix} x \\ \alpha \end{pmatrix}$, that describes the laser beam just after reflection from a mirror at an angle of θ (where θ is defined as the angle between the mirror and the optical axis as shown in the figure above). Assume that the value of θ is not far from 45 degrees. *[5 marks]*

(iv) Calculate an expression that relates the position that the light will hit the retina in terms of the angle of the mirror, θ, and the characteristics of the eye. *[6 marks]*

(v) If the spacing between the mirror and the front surface of the eye is 20cm, the radius of curvature of the front surface of the eye is 6mm, the refractive index of the material in the eye is 1.5 and the spacing between front and back surfaces of the eye is 2.6cm, then calculate the required angle to hit a point on the retina 1mm from the optical axis *[5 marks]*

(vi) Imagine that a lens is inserted into the optical path a few cm before the rotating mirror in order to focus the laser onto the retina. Without calculation, consider what focal length this lens would need to have in order to focus a beam onto the retina i.e. will the focal length be the same, longer than, or less than physical distance between the inserted lens and the back of the eye? *[5 marks]*

2. (a) Lasers based on a ring of optical fibre that has been doped with rare-earth ions are becoming increasingly popular. An example of such a laser is shown in the figure below. An 4-port fibre coupler is inserted into the loop and functions much as a partially reflective mirror would in normal laser: it lets a fraction of the laser light incident from one side across to its other side, and vice-versa. In particular, a small fraction of the light incident on port A will cross-over and appear out of port D while the rest continues on to appear in port C. In addition, a small fraction of the light incident on port B will cross-over and appear in port C, while the rest continues on to port D. In this example we have chosen a coupler that allows 1% of the laser light to cross-over in both cases i.e. 1% of the light incident on ports A and B ends up in D and C respectively. The refractive index of the fibre ring is 1.7, and for part (a) of the question, the external pump that creates optical gain in the loop is off.



A single short pulse of light at a wavelength of 1.5μm (less than 1ns in duration) is injected into port A of the coupler. One observes a stream of pulses emitted from port C with a time between pulses of 57ns. The energy of the output pulses is seen to decrease to 1/e of the first pulse's value over a time of 2.83 microseconds.

(i) Calculate the physical length of the fibre ring. *[4 marks]*

(ii) Is the only loss in the cavity that which is associated with the coupler transmission? Calculate the value of the additional loss, if any, in units of per unit length. *[6 marks]*

(iii) Calculate the cavity Q of the fibre ring from these observations. *[4 marks]*

(iv) Calculate the Finesse of the fibre ring from these observations *[4 marks]*

(b) Imagine that the rare-earth doped fibre ring is now pumped with an intense external source of radiation at 980nm. A pump at this wavelength can produce optical gain within the ring via a population inversion of the rare earth ion system. The rare-earth ion system can be considered a good approximation to a 4 level laser operating at an output wavelength of 1.5μm.

(i) Calculate the minimum gain (in units of m^{-1}) required from the fibre ring to make this ring spontaneously oscillate? *HINT: If you can't do question 2(a) then assume an additional loss of 0.1*m^{-1} *and continue on in the question* *[4 marks]*

(ii) If the pumped fibre laser is capable of providing optical gain over 100GHz of spectrum centred at a wavelength of 1.5μm how many axial modes might oscillate? Would this make a good continuous wave laser? *[4 marks]*

(iii) What is the threshold pump power for laser oscillation? You might find it useful to consider that the spontaneous lifetime of upper laser state is around 1microsecond, while the material is capable of delivering gain over approximately 100 GHz of the spectrum. *Hint: Calculate the value of the line broadening function at the centre of the gain curve, and from this calculate the threshold population inversion density required for laser oscillation. Use the usual rectangular shape approximation of the line broadening function, g(ν), to simplify the problem.* *[7 marks]*

3.(a)(i) A Gaussian-spherical laser beam with complex radius of curvature q(z) evolves as a function of distance as $q(z) = q_0+z$, where z is measured from the waist point, and q_0 is the complex radius of curvature at the waist. If R(z) is the radius of curvature of the beam, and w(z) is the beam spot size (definition: radius where amplitude has fallen to 1/e of its central value) then the complex radius of curvature is defined as:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi w(z)^2}$$

where $j = \sqrt{-1}$. By manipulation of this expression one can find that the beam spot size evolves as:

$$w(z) = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2}$$

and the beam radius of curvature evolves as:

$$R(z) = z + \frac{z_R^2}{z}$$

where the Rayleigh range, z_R, is defined as:

$$z_R = \frac{\pi w_0^2}{\lambda}$$

Using these expressions, design a single lens optical system that can focus a collimated laser beam to a waist size of 10μm. The input laser beam is delivered by a Nd:YAG laser with an output wavelength of 532nm and a beam spot size of 1 cm at the lens. Determine an appropriate focal length for the lens. You might find it useful to be reminded that lenses transform an input Gaussian-spherical beam with a radius of curvature, R_{old}, into a new radius of curvature, R_{new}, via the transformation:

$$R_{\text{new}} = \frac{f R_{\text{old}}}{f - R_{\text{old}}}$$

[12 marks]

(b) A certain two-level atomic system (with energy levels E_1 and E_2) is in thermal equilibrium at 300K. The ratio of the lower state population density (denoted n_1), and upper state population density (denoted n_2), for this system is $n_1/ n_2 = e \sim 2.718\ldots$. Use the Einstein A_{21}, B_{21} and B_{12} coefficients to denote the probability of spontaneous transition and interactions with the radiation field respectively.

(i) When broadband light is incident on this system, what is the frequency (in Hz) of the photons absorbed or emitted by the system? *[4 marks]*

(ii) Write down the rate equations of the system if it is being exposed to broadband radiation that has an energy density of u (in J/m^3) *[8 marks]*

(iii) From the rate equations found in part (ii) calculate the ratio of populations, n_1/ n_2, as a function of u and the parameters of the atomic system (A_{21}, B_{21} and B_{12}). *[6 marks]*

(iv) If the atomic system is pumped to $n_1/ n_2 = 1.1$ calculate the radiation density, as well as the ratio of rates of spontaneous and stimulated emission processes under these conditions

[6 marks]

END OF PAPER