(see over)

<u>Potentially Useful Formulae and Constants:</u>

Speed of light, c= 3 x 10^{8} ms^{-1}
Charge of the Electron, e = 1.6 x 10^{-19} C
Planck's Constant, h = 6.6 x 10^{-34} J s
Boltzmann's constant, k = 1.38 x 10^{-23} J K^{-1}

(The symbols below have their usual meaning)

Resonator g factors: $g_1 = 1- L/R_1$, $g_2 = 1-L/R_2$

ABCD matrices for a curved mirror, thin lens, propagation, and a spherical dielectric interface:

$$\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}, \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ \dfrac{n_2 - n_1}{n_2 R} & n_1/n_2 \end{bmatrix}.$$

Finesse of a resonator with no loss apart from that associated with the finite reflectivity of the two mirrors. The mirror reflectivity is expressed in terms of its amplitude reflectivity:

$$F = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

Planck's radiation law: $u = \dfrac{8\pi h\nu^3}{c^3\left(e^{h\nu/(kT)} - 1\right)}$

Ratio of population densities in two levels separated by ΔE in energy: $n_2/n_1 = e^{-\Delta E/(kT)}$

Q of a cavity resonance mode: $Q_c = \nu_0/\Delta\nu$ or $Q_c=2\square\nu_0$ (stored energy/ energy dissipated per second) or $Q_c= 2\square$ (stored energy/ energy dissipated per cycle)

Gain coefficient for a 4 level scheme in terms of population differences: $\kappa = \dfrac{c^2 g(\nu_r)A}{8\pi\nu_r^{\,2}}(n_3 - n_2)$

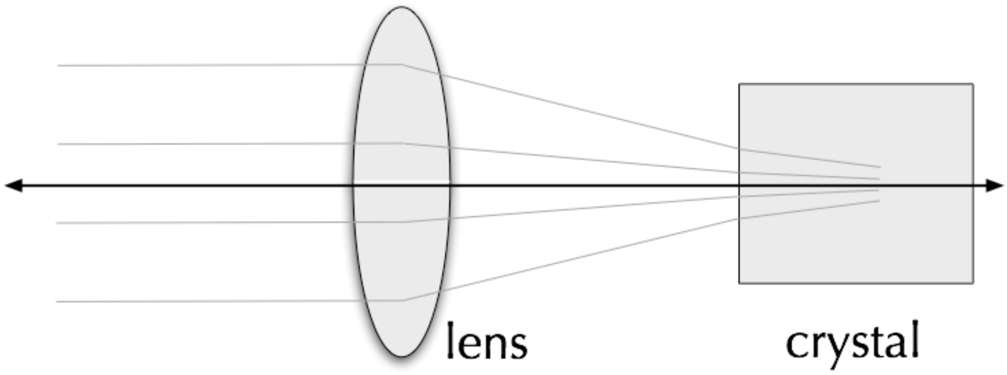
Population inversion at threshold for a 4 level laser scheme: $(n_3 - n_2)/N = W_{41}/(W_{41} + A_{32})$

(see over)

1.(a)(i) Can an optical resonator with two convex mirrors be optically stable? *[1 mark]*

(ii) Can an optical resonator with one convex and one concave mirror be optically stable? *[1 mark]*

1.(b) One frequently encountered situation in modern optics laboratories is the need to focus a beam of laser radiation into the exact centre of an optical crystal. This crystal might be designed for generating non-linear radiation, or modulating the phase, frequency or power of the optical signal. A figurative view of this situation is shown below:



Assume that the laser beam is perfectly collimated at the input lens, that the lens can be adequately described by a thin lens approximation, that the refractive index of air is 1, and that the input crystal face is both flat and normal to the optical axis of the system.

(i) Write down a sequence of ray matrices that describe the path of a ray that starts at the input face of the lens and on to an arbitrary point inside the crystal. *[6 marks]*

(ii) Simplify the ray matrix sequence you found in the part (i) into a one matrix *[4 marks]*

(iii) Imagine that a 10 cm focal length lens is used to focus the laser beam into a 2 cm long crystal with a refractive index of 2.2. Calculate the required spacing of the lens from the input face of the crystal so that the focal point of the input radiation will be at the exact mid-point of the crystal along the optical axis. *[8 marks]*

(iv) Imagine that the crystal was birefringent and demonstrated a refractive index of 2.2 for radiation polarized in the plane of the page, and refractive index of 2.3 for radiation polarized perpendicular to the page. Calculate the difference in the focal point positions for the two polarizations. Assume that there is no birefringence in the lens itself. *[7 marks]*

(v) If the crystal were to be removed calculate in which direction the focal point would move, and by what amount when compared with the focal point calculated in question (iii) above *[6 marks]*

2.(a) Semiconductor lasers are often fabricated from a single doped semiconductor crystal in which two parallel surfaces are created by cleaving along its natural crystal planes. These surfaces act as reflectors and thereby serve as the resonator mirrors for the laser without the need for any additional high reflection coating. The power reflectivity, R, for such a dielectric surface at normal incidence is:

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

where n_1 and n_2 are the dielectric constant of the two materials on either side of the dielectric surface.

(i) If the semiconductor laser crystal has a refractive index of $n_2 \sim 3.6$ and it is placed in air ($n_1 \sim 1$) then calculate the reflectivity of the mirrors. Assume that crystal surfaces are normal to the incident light *[2 marks]*

A single short pulse of light at 850nm (less than 1ps in duration) is injected through one resonator mirror. One observes a stream of pulses transmitting through the other mirror with a time between pulses of 100ps. The energy of the output pulses is seen to decrease to 10^{-6} of its initial value over a time of 400ps.

(ii) Calculate the physical distance between the two cleaved surfaces that are acting as the resonator mirrors. *[5 marks]*

(iii) Is the only loss in the cavity due to the finite reflectivity of the mirrors? Calculate the value of the additional loss, if any, (in units of per unit length) of the semiconductor material? *[6 marks]*

(iv) Calculate the crystal cavity Q from these observations. *[6 marks]*

(b) If we now connect the semiconductor crystal, in the right way, to a source of electrical energy it is possible to produce optical gain within the crystal via a population inversion.

(i) Calculate the minimum gain (in units of m^{-1}) required from the electrically excited semiconductor crystal to make this resonator oscillate? *HINT: If you cannot do question 2(a) then assume an arbitrary loss and continue on in the question* *[6 marks]*

(ii) If the semiconductor crystal is capable of providing optical gain over 10GHz of spectrum centred at a wavelength of 850nm how many axial modes might oscillate? Would this make a good laser? *[4 marks]*

(iii) The semiconductor laser can be considered a good approximation to a 4 level laser system operating at an output wavelength of 850nm. Additionally you might find it useful to consider that the spontaneous lifetime of upper laser state is around 1ps, while the material is capable of delivering gain over approximately 10 GHz of the spectrum.

Calculate the value of the line broadening function at the centre of the gain curve, and from this calculate the threshold population inversion density required for laser oscillation. Use the usual rectangular shape approximation of the line broadening function, g(ν), to simplify the problem. *[4 marks]*