

- 3.(a)(i) A Gaussian-spherical laser beam with complex radius of curvature $q(z)$ evolves as a function of distance as $q(z) = q_0 + z$, where z is measured from the waist point, and q_0 is the complex radius of curvature at the waist. If $R(z)$ is the radius of curvature of the beam, and $w(z)$ is the beam spot size then the complex radius of curvature is defined as:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j \frac{\lambda}{\pi w(z)^2}$$

where $j = \sqrt{-1}$. By manipulation of this expression one can find that the beam spot size evolves as:

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}$$

and the beam radius of curvature evolves as:

$$R(z) = z + \frac{z_R^2}{z}$$

where the Rayleigh range, z_R , is defined as:

$$z_R = \frac{\pi w_0^2}{\lambda}$$

Using these expressions, design a single lens optical system that can focus a collimated laser beam to a waist size of $20\mu\text{m}$. The input laser beam is delivered by a Nd:YAG laser with an output wavelength of 532nm and a beam spot size of 1 cm at the lens. Give me an appropriate focal length lens and its position with respect to the highly focussed waist. You might find it useful to be reminded that lenses transform Gaussian-spherical beams with a input radius of curvature, R_{old} , into a new radius of curvature, R_{new} , via the transformation:

$$R_{\text{new}} = \frac{f R_{\text{old}}}{f - R_{\text{old}}}$$

[12 marks]

- (b)(i) Write down the rate equations for each of the levels of a two level atomic system (with energy levels E_1 and E_2) in equilibrium with a radiation field having an energy density of u ($\text{J}\cdot\text{m}^{-3}$) at the transition frequency $\nu_r = (E_2 - E_1)/h$. Use the Einstein A_{21} , B_{21} and B_{12} coefficients to denote the probability of transition and the interactions with the radiation field. Denote the number density of the lower state with n_1 , and of the upper state with n_2 .

[8 marks]

- (ii) If the radiation field in question (i) above is actually black-body radiation (still with a radiation density of u), and the atomic system is in thermal equilibrium with this radiation, then calculate the relationships between the Einstein A_{21} , B_{21} and B_{12} coefficients in terms of the transition frequency and other constants.

[6 marks]

- (iii) If the radiation field in question (i) above is a strong laser signal tuned to the transition frequency, $\nu_r = (E_2 - E_1)/h$ (still with energy density of u), then calculate the energy density required to keep $1/3$ of the atoms in a steady-state in the excited state.

[8 marks]

END OF PAPER

