

Potentially Useful Formulae and Constants:

Speed of light, $c = 3 \times 10^8 \text{ ms}^{-1}$
Charge of the Electron, $e = 1.6 \times 10^{-19} \text{ C}$
Planck's Constant, $h = 6.6 \times 10^{-34} \text{ J s}$
Boltzmann's constant, $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$

(The symbols below have their usual meaning)

Resonator g factors: $g_1 = 1 - L/R_1, \quad g_2 = 1 - L/R_2$

ABCD Matrices for a curved mirror, thin lens, propagation, and a spherical dielectric interface:

$$\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{n_2 - n_1}{n_2 R} & n_1/n_2 \end{bmatrix}$$

Finesse of a resonator with two mirrors, each having an amplitude reflectivity of r: $F = \pi r / (1 - r^2)$

Planck's radiation law: $u = \frac{8\pi h \nu^3}{c^3 (e^{h\nu/(kT)} - 1)}$

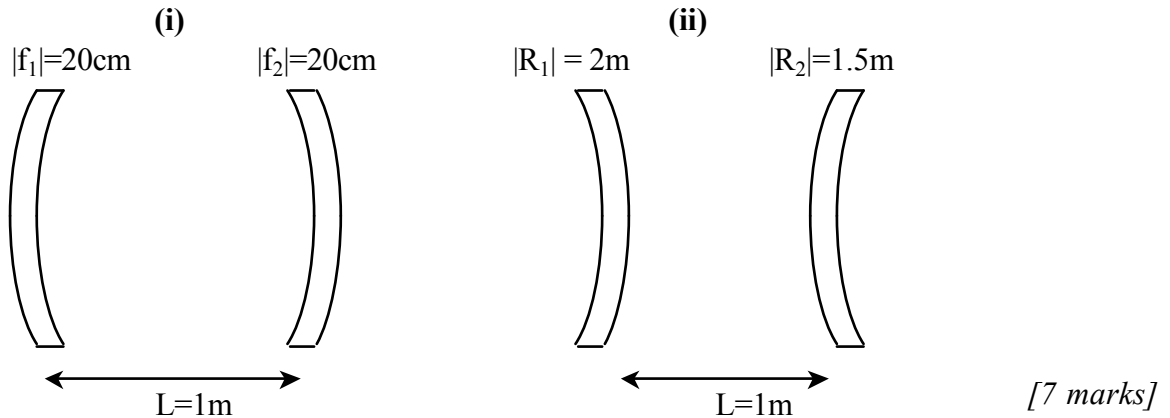
Ratio of populations densities in two levels separated by ΔE in energy: $n_2 / n_1 = e^{-\Delta E / (kT)}$

Q of a cavity resonance mode: $Q_c = \nu_0 / \Delta \nu$ or $Q_c = 2\pi \nu_0$ (stored energy/ energy dissipated per second)
 or $Q_c = 2\pi$ (stored energy/ energy dissipated per cycle)

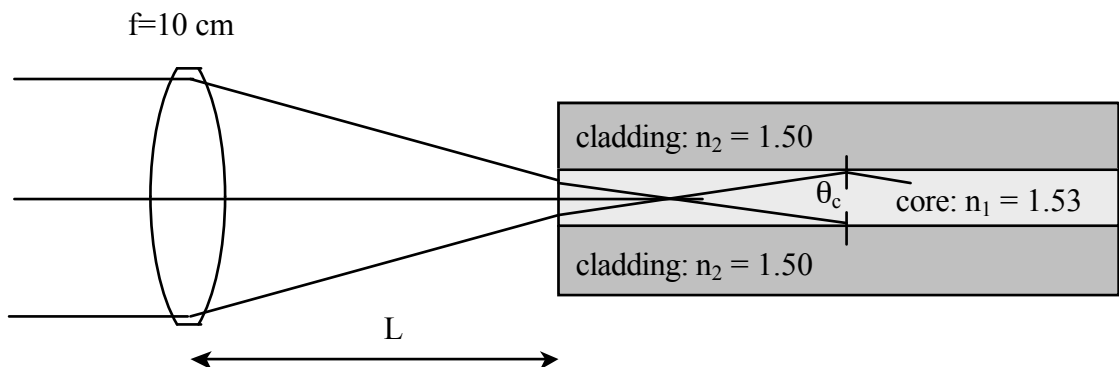
Gain coefficient for 4 level scheme in terms of population differences: $\kappa = \frac{c^2 g(\nu_r) A}{8\pi \nu_r^2} (n_3 - n_2)$

Population inversion at threshold for four level laser scheme: $(n_3 - n_2) / N = W_{41} / (W_{41} + A_{32})$

- 1.(a) Determine whether either of the following two resonators can support stable optical modes. R indicates the radius of curvature of the associated mirror, f indicates a mirror's focal length, and L is the spacing between the mirrors. If you can change the length of the resonators (while leaving the mirror curvatures the same) is it possible to bring each of the resonators into a stable range? Specify the range of resonator lengths, if any, that makes each of the resonators stable.



- (b) Consider an optical system consisting of a thin lens, with a focal length of 10cm, focussing a 1cm diameter collimated light beam into a multimode optical fibre. The optical fibre consists of a 200 μm diameter high refractive index core surrounded by a lower refractive index cladding. A ray optics approach is valid for this problem.



- (i) Write down a ray matrix which describes the path of the light from the lens to the interior of the fibre. [7 marks]
- (ii) What restrictions can be placed on the lens-fibre distance, L , so that all of the incoming beam of light will be focussed into the core? [6 marks]
- (iii) In order for the light that is focussed into the core of the fibre to be guided along the core, it is necessary for the angle that the light makes with the normal to the core-cladding interface to be greater than the critical angle, $\sin \theta_c = n_2/n_1$. Using the material values given in the diagram above calculate the maximum possible diameter of the collimated beam which satisfies this requirement. [5 marks]

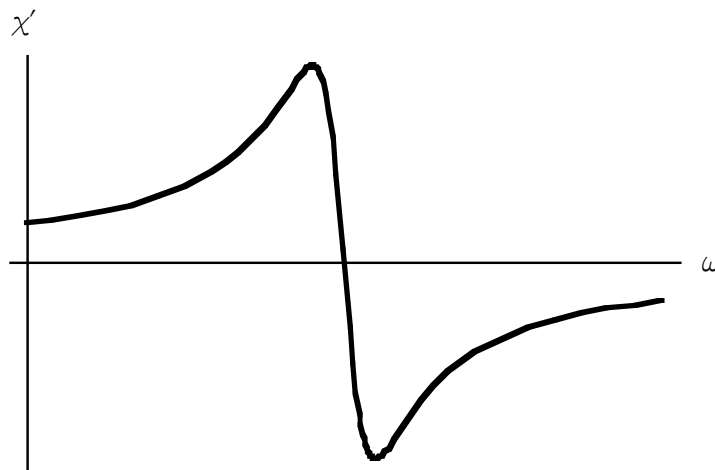
- 2.(a) The temporal width of a light pulse has a measured full width at half maximum (FWHM) of 50 fs. Assuming a transform-limited Gaussian pulse shape and a laser cavity length of 30cm, how many frequency modes are contained in the FWHM of the power spectrum? The following equations may be helpful:

$$E(t) = e^{-at^2} e^{-i(bt^2 + \omega_0 t)} \quad (1)$$

$$\Delta f_p \cdot \tau_p = \frac{2 \ln(2)}{\pi} \sqrt{1 + (b/a)^2} \quad (2)$$

Equation (1) describes the electric field behaviour of the light pulse with carrier frequency ω_0 while Equation (2) represents the time-bandwidth-product in terms of the spectral and temporal FWHM of a light pulse. [5 marks]

- (b) A medium contains an atomic transition with the real part of its complex susceptibility showing the behaviour given in the figure below:



Given that expressions for phase velocity and group velocity in the medium are given by

$v_\phi(\omega) = \frac{c}{1 + \chi'(\omega)/2}$ and $v_g(\omega) = \frac{v_\phi(\omega)}{1 - (\omega/v_\phi) dv_\phi/d\omega}$ respectively, explain whether or not the medium will support group velocities faster than the speed light. [3 marks]

- (c) The width of a short light pulse, σ_0 , as it propagates along a distance z , may be expressed as:

$$\sigma(z)/\sigma_0 = \left[\left(1 + \frac{C\beta_2 z}{2\sigma_0^2} \right)^2 + \left(\frac{\beta_2 z}{2\sigma_0^2} \right)^2 + \left(\frac{1+C^2}{8} \right)^2 \left(\frac{\beta_3 z}{2\sigma_0^3} \right)^2 \right]^{1/2}$$

for Gaussian shaped pulses, where σ_0 is the initial pulse width, C is the chirp parameter, and β_2 and β_3 are the second and third order dispersion parameters respectively. By examination of this equation explain the important difference between the effect of β_2 and β_3 in relation to chirp.

An optical communication system is operating with transform-limited Gaussian input pulses and zero third-order dispersion fibre. Find the optimum initial pulse width in terms of β_2 and z which will minimize pulse broadening in the fibre. [6 marks]

- (d) Estimate the optical intensity that may be expected to cause an optical medium into which it is injected to produce significant phase shifting of the light (e.g. half a wavelength). Choose a wavelength of 800 nm, a crystal length 10mm and assume $n_2 = 3 \times 10^{-16} \text{ cm}^2/\text{W}$.

If the output power of a 1GHz repetition-rate mode-locked laser is 500mW decide whether the light pulses in the laser cavity with finesse 100 will be sufficient to produce strong levels of self phase modulation. Assume the optical pulse width to be 40fs and a waist with radius $10\mu\text{m}$ in the lasing medium. *[6 marks]*

- (e) State the characteristics of a mode-locked laser that allow it to produce extremely short pulses. For example, what factors play a dominant role in keeping the pulse short as the pulse propagates around a laser cavity. *[3 marks]*

- (f) What are two advantages of an interferometric autocorrelator in comparison to an intensity autocorrelator? *[2 marks]*

- 3.(a)(i) Consider a Fabry-Perot cavity made of two identical mirrors spaced apart by a distance L . The cavity was illuminated by a 100ps pulse from an external 1064nm laser source. The output beam of the cavity is observed to be a regular sequence of 100ps pulses spaced apart by 1ns. The energy of the output pulses decreases exponentially with a time constant of 100ns. Calculate the cavity length, the cavity finesse, the mirror reflectivity, and the cavity Q from this observation. *[6 marks]*

- (ii) The interior of the cavity is now filled with a Nd:YAG crystal with a refractive index equal to 1 and an absorption coefficient equal to 0.03m^{-1} (both values are quoted for a wavelength of 1064nm). The Nd:YAG crystal is **not** being pumped by any external source in order to produce population inversion. What will be the new values of the finesse, cavity Q and cavity photon lifetime? *[6 marks]*

- (iii) What would be the minimum gain (in units of m^{-1}) required from the Nd:YAG crystal in order to cause the filled resonator to oscillate? *[3 marks]*

- (iv) If the Nd:YAG crystal is capable of providing optical gain over 12GHz of spectrum centred on 1064nm how many axial modes could possibly oscillate? *[2 marks]*

- (v) If a Nd:YAG laser crystal has the following parameters
spontaneous lifetime of upper laser state $\sim 0.23\text{ms}$, pump frequency $\sim 6.25 \times 10^{14} \text{ Hz}$,
density of Nd ions $\sim 1.6 \times 10^{19} \text{ cm}^{-3}$.

Give the value of the line broadening function at the centre of the gain curve, and from this calculate the threshold population inversion density required for laser oscillation. Use the usual rectangular shape approximation of the line broadening function, $g(\nu)$, to simplify the problem. *[4 marks]*

- (vi) Calculate the threshold pump power density required to achieve laser oscillation. *[4 marks]*