4. (a)(i) A Gaussian-spherical beam with complex radius of curvature q(z) evolves as a function of distance as $q(z) = q_0+z$, where z is measured from the waist point, and q_0 is the complex radius of curvature at the waist. If R(z) is the radius of curvature of the beam, and w(z) is the beam spot size then the complex radius of curvature is defined as:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j\frac{\lambda}{\pi w(z)^2}$$

where $j = \sqrt{-1}$. Using the expressions given above find an expression for the point at which the beam diameter has expanded to a factor of 2 larger than its minimum value. Express your answer in terms of the beam waist size and the wavelength of the radiation. *[6 marks]*

(ii) What is the radius of curvature of the beam at this point? *[4 marks]*

(b)(i) Write down the rate equations for each of the levels of a two level atomic system (with energy levels E_1 and E_2) in equilibrium with a radiation field having an energy density of u Jm^{-3} at the transition frequency $\nu_r = (E_2-E_1)/h$. Denote the number density of the lower state with n_1, and of the upper state with n_2. *[6 marks]*

(ii) If the radiation field is black-body radiation and the atomic system is in thermal equilibrium with this radiation field, calculate a relationship between the Einstein A and B coefficients in terms of the transition frequency. *[6 marks]*

(iii) What is the ratio of the rates of spontaneous and stimulated emission from the upper level in a two level atomic system in thermal equilibrium with blackbody radiation? At what transition wavelength will the ratio be 1 for a system bathed in a 300K black body radiation field? *[3 marks]*

END OF PAPER