(see over)

Potentially Useful Formulae and Constants:

Speed of light, c= 3 x 10^8 ms^{-1}
Charge of the Electron, e = 1.6 x 10^{-19} C
Planck's Constant, h = 6.6 x 10^{-34} J s
Boltzmann's constant, k = 1.38 x 10^{-23} J K^{-1}

(The symbols below have their usual meaning)

Resonator g factors: $g_1 = 1- L/R_1$, $g_2= 1-L/R_2$

ABCD Matrices for a curved mirror, thin lens, propagation, and a spherical dielectric interface:

$$\begin{bmatrix} 1 & 0 \\ -2/R & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}, \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ \dfrac{n_2 - n_1}{n_2 R} & n_1/n_2 \end{bmatrix}.$$

Finesse of a resonator with two mirrors, one having an amplitude reflectivity of r_1, while the second has an amplitude reflectivity of r_2:

$$F = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$$

Planck's radiation law: $u = \dfrac{8\pi h\nu^3}{c^3\left(e^{h\nu/(kT)} - 1\right)}$

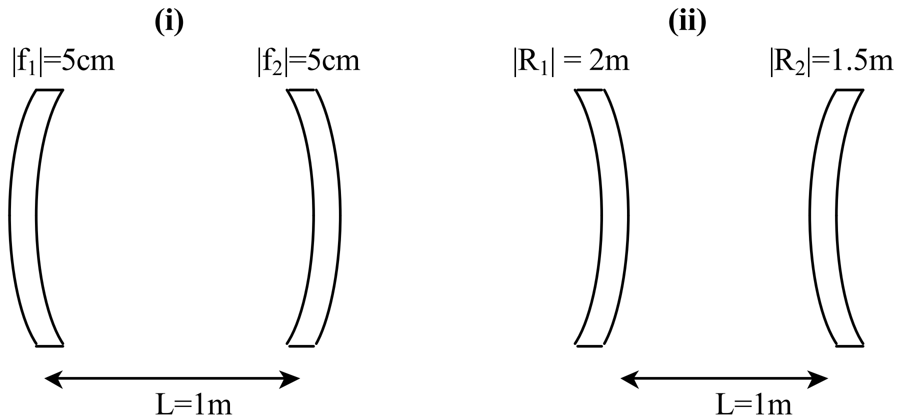
Ratio of population densities in two levels separated by ΔE in energy: $n_2/n_1 = e^{-\Delta E/(kT)}$

Q of a cavity resonance mode: $Q_c = \nu_0/\Delta\nu$ or $Q_c=2\pi\nu_0$ (stored energy/ energy dissipated per second) or $Q_c=2\pi$ (stored energy/ energy dissipated per cycle)

Gain coefficient for 4 level scheme in terms of population differences: $\kappa = \dfrac{c^2 g(\nu_r) A}{8\pi\nu_r^2}(n_3 - n_2)$

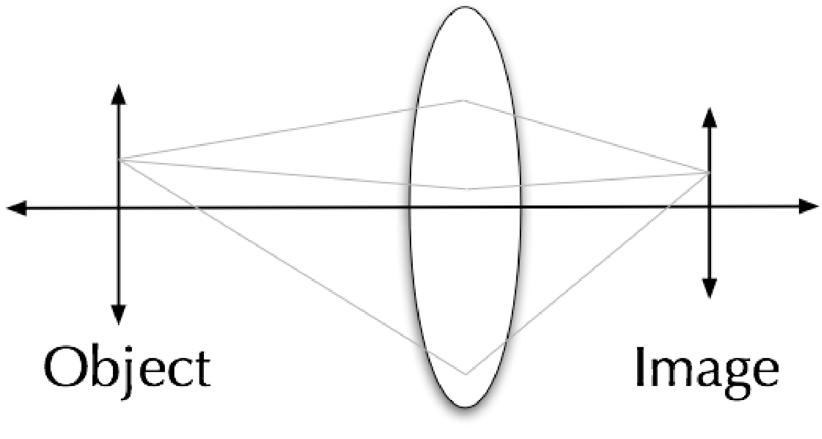
Population inversion at threshold for four level laser scheme: $(n_3 - n_2)/N = W_{41}/(W_{41} + A_{32})$

1.(a) Determine whether either of the following two resonators can support stable optical modes. R indicates the radius of curvature of the associated mirror, f indicates a mirror's focal length, and L is the spacing between the mirrors. If you can change the length of the resonators (while leaving the mirror curvatures the same) is it possible to bring the resonators into a stable range? Specify the range of resonator lengths, if any, that makes each of the resonators stable.



[6 marks]

(b) When building an imaging system, the aim is for each point on the object to map to a single point in the image plane. In other words, each light ray leaving a particular point on the object should map to a single point in the image, independent of its initial slope.



(i) Write down a ray-matrix expression that describes the path of a light ray starting on an object, through a lens, and then onto a plane some arbitrary distance beyond the lens. *[7 marks]*

(ii) Find an expression for the position of a suitable image plane in terms of both the object distance, and the focal length of the lens. *Hint: impose the condition mentioned at the start of this question for an imaging system.* *[8 marks]*

(iii) The magnification of an imaging system is defined as the ratio of the image to the object size. Using the expression derived in (i), show that the magnification of the system is equal in magnitude to the ratio of the image to object distance. *[7 marks]*

(iv) Calculate the image distance and magnification of the optical system for an object 10cm from a 5cm focal length convex lens. *[5 marks]*

2. (a)(i) Consider a Fabry-Perot cavity made of two mirrors spaced apart by a distance L. One mirror is considered to have 100% reflectivity. The cavity is illuminated through the other mirror by a 50ps pulse from an external 1064nm laser source. The output beam of the cavity is seen to be a regular sequence of 50ps pulses spaced apart by 1ns. The energy of the output pulses decreases exponentially with a time constant of 100ns. Calculate the cavity length, the cavity finesse, the mirror reflectivity, and the cavity Q from this observation. *[8 marks]*

(ii) Imagine that the interior of the cavity is now filled with a Nd:YAG crystal with a refractive index equal to 1 and an absorption coefficient equal to 0.03m^{-1} (both values are quoted for a wavelength of 1064nm). The Nd:YAG crystal is **<u>not</u>** being pumped by any external source in order to produce population inversion. What will be the new values of the finesse, cavity Q and cavity photon lifetime? *[8 marks]*

(iii) If we now decided to pump the Nd:YAG crystal so that it would produce optical gain, calculate the minimum gain (in units of m^{-1}) required from the Nd:YAG crystal in order to make this filled resonator start to oscillate? *HINT: If you cannot do part (ii) then assume the loss value from part (i) and continue on in the question* *[5 marks]*

(iv) If the Nd:YAG crystal is capable of providing optical gain over 6 GHz of spectrum centred on 1064nm how many axial modes might oscillate? Would this make a good laser? *[4 marks]*

(v) If a Nd:YAG laser crystal has the following parameters:
spontaneous lifetime of upper laser state ~ 0.23ms, pump frequency ~ 6.25×10^{14} Hz, density of Nd ions ~ $1.6 \times 10^{19}\ \text{cm}^{-3}$, the gain is broadened over 6 GHz of the spectrum. Nd:YAG provides a four level laser system.

Calculate the value of the line broadening function at the centre of the gain curve, and from this calculate the threshold population inversion density required for laser oscillation. Use the usual rectangular shape approximation of the line broadening function, $g(\nu)$, to simplify the problem. *[4 marks]*

(vi) Calculate the threshold pump power density required to achieve laser oscillation. Is this an efficient laser system suitable for continuous wave operation? *[4 marks]*

(see over)