

- 3.(a)(i) A Gaussian-spherical beam with complex radius of curvature $q(z)$ evolves as a function of distance as $q(z) = q_0 + z$, where z is measured from the waist point, and q_0 is the complex radius of curvature at the waist. If $R(z)$ is the radius of curvature of the beam, and $w(z)$ is the beam spot size then the complex radius of curvature is defined as:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - j \frac{\lambda}{\pi w(z)^2}$$

where $j = \sqrt{-1}$. Using the expressions given above find an expression for the point at which the beam diameter has expanded to a factor of 4 larger than its minimum value. Express your answer in terms of the beam waist size and the wavelength of the radiation. *[8 marks]*

- (b)(i) Write down the rate equations for each of the levels of a two level atomic system (with energy levels E_1 and E_2) in equilibrium with a radiation field having an energy density of u (J.m^{-3}) at the transition frequency $\nu_r = (E_2 - E_1)/h$. Denote the number density of the lower state with n_1 , and of the upper state with n_2 . *[8 marks]*
- (ii) If the radiation field is black-body radiation and the atomic system is in thermal equilibrium with this radiation field, calculate a relationship between the Einstein A and B coefficients in terms of the transition frequency. *[8 marks]*
- (iii) If the radiation field is a strong laser signal tuned to the transition frequency, $\nu_r = (E_2 - E_1)/h$, calculate the energy density required to keep 1/3 of the atoms in the excited state. *[10 marks]*

END OF PAPER