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Assignment 2: Gaussian beams and mode-matching

1. To excite just the fundamental axial (or equivalently the lowest order transverse) mode of a resonator, it is necessary to match an incident Gaussian beam's spot size and curvature to that of the fundamental cavity mode. This is usually done in two steps: (1) find the location in space at which the spot size of the fundamental mode (propagated forward) and the laser beam spot size (propagated backward) are identical, and then (2) calculate which thin lens will transform the incident beam's curvature so that it matches that of the fundamental mode at that location.
 - (a) Imagine a 1 metre long resonator with two concave mirrors of focal lengths of 93 cm and 1.5 metre. Both mirrors have a power reflectivity of 96%. Is this resonator stable?
 - (b) Calculate the parameters of the fundamental Gaussian mode for this resonator at 850 nm. Where is the beam waist situated in the cavity? What are the beam spot sizes at the mirrors?
 - (c) Imagine a laser beam (wavelength = 830nm) generated by a laser that has a 88cm long symmetric cavity. The beam has a waist size of $120\mu\text{m}$. If the laser output mirror is 3 metres from the 93cm mirror and 4m from the 1.5m mirror of the resonator mentioned above, calculate at which point(s) the laser beam spot size, and the spot size of the fundamental mode in the resonator are the same. Remember that if the beam has propagated a long or short distance compared with its Rayleigh range there may be a simplification that will make this calculation a lot simpler.
 - (d) Imagine a conventional spherical wavefront (not a gaussian-spherical beam). An equivalent representation of this wavefront can be given using its propagation rays i.e. the rays that are everywhere perpendicular to the wavefront surface. You can find this approach described more fully in the lecture 5 overheads. Using this ray matrix/vector notation, write an expression for a spherical wavefront, of radius of curvature R , in terms of its propagation rays (Use the paraxial approximation: we are only interested in the parts of the wavefront that are near the optical axis). Make use of ray matrices to calculate the effect a thin lens has on the radius of curvature of a spherical beam passing through the lens. As mentioned in the lectures, this approach can also be applied to determine the change in curvature of a Gaussian-spherical beam resulting from passing through a thin lens. Hence find the characteristics of a thin lens, or mirror, that could be used to match the laser beam parameters to the resonator parameters at the point calculated in part (c).
2. Consider a Fabry-Perot cavity of two identical mirrors separated by a distance L which contains some relatively low-loss gaseous material between the mirrors. This cavity is illuminated by a 1ns pulse from an external 532nm laser source. The output beam of the cavity is observed to be a regular sequence of 1 ns pulses spaced apart by 5ns. The energy of the output pulses decreases exponentially with a time constant of 35ns. If the refractive index of the gas is 1.03 and its loss is 0.03m^{-1} then calculate the cavity length, the cavity finesse, the mirror reflectivity, photon lifetime, and cavity Q from this single observation.
3. Imagine you wish to construct a laser-based communication link between Kalamunda and Rottnest (approximate distance 50km). You have a so-called "eye-safe" $1.5\mu\text{m}$ wavelength laser source with an output power of 100mW that you will project using a 30cm diameter telescope system. The receiver system is based on a 3cm telescope system and a photodiode with a $1.5\mu\text{m}$ optical filter in front of it. Assume that to effectively emit the radiation it is necessary for the telescope mirror radius to be three times larger than the spot-size of the beam (what would happen if this wasn't the case?). In addition, assume that there is a reduction in intensity by a factor of $1/e$ every 50 kilometres due to absorption by the air at this wavelength. Try and make all reasonable assumptions.
 - (a) Calculate the approximate power collected at Rottnest.