

Statistical Mechanics Problems II

(Assessed as extra credit at a value of 3% of total course mark. Due 12 September, 2008. **Hand in to Simon Tyler!**)

1. Questions on Brownian Motion.

(a) W. Pospisil observed the Brownian motion of soot particles of radius 0.4×10^{-4} cm. The particles were immersed in a water-glycerin solution having viscosity 0.0278 gm/(cm s) at a temperature of 18.8°C . The observed mean square x component of the displacement in a 10 second time interval was 3.3×10^{-8} cm². Calculate Avogadro's number using this data.

(b) Consider again the Langevin equation as written above.

i) Write $F(t)$ and v in terms of Fourier integrals, and show how their Fourier coefficients must be related to satisfy the Langevin equation. Express the spectral density of the velocity in terms of the spectral density of the force $F(t)$.

ii) Use the Wiener-Khintchine relations to find the spectral density of the velocity v from the correlation function

$$\langle v(0)v(s) \rangle = \langle v^2(0) \rangle e^{-\gamma|s|} = \frac{k_B T}{m} e^{-\gamma|s|}.$$

iii) Derive Nyquist's theorem by combining the preceding results to find an explicit expression for the spectral density of the force $F(t)$ in terms of γ .

2. Questions on fluctuations.

(a) A string of length l is stretched, under a constant tension F , between two fixed points A and B . Show that the mean square value of the fluctuational displacement $y(x)$ at the point P , a distance x from A , is given by

$$\langle y(x)^2 \rangle = \frac{k_B T}{Fl} x(l-x).$$

Hint: Evaluate the energy E associated with the fluctuations in question. The relevant probability distribution is then given by $p \propto \exp(-E/k_B T)$. This distribution can be used to give the required average.

(b) The correlation function $K(s)$ of a statistically stationary variable $y(t)$ can be given by

$$K(s) = K(0)e^{-\alpha^2 s^2} \cos(\omega s)$$

where $\alpha > 0$. Find the power spectrum. Discuss the behaviour of, and give physical interpretations for, the power spectra in the limits

i) α goes to 0

ii) ω goes to 0

iii) ω and α both go to zero.