to obtain

$$\frac{d}{dt}\rho_m = \lambda \sum_n \left(\rho_n e^{\beta E_n} - \rho_m e^{\beta E_m} \right).$$

dimensions of time^{-1}

$\Rightarrow$ λ^{-1} represents time for decay due to heat bath

Example: Magnetic Resonance:

Suppose a system of spin $\frac{1}{2}$ particles (could be electrons or neutrons) each with magnetic moment μ.

Apply magnetic field H:

$$E_{\pm} = \mp \mu H \quad \text{for } \binom{\text{parallel}}{\text{anti-parallel}} \text{ to } H$$

$$\underset{+\mu}{\uparrow} \quad \underset{-\mu}{\downarrow} \qquad \Uparrow H$$

Define state occupation as

$$\rho_n \rightarrow \begin{cases} n_+ & \text{up spins } (+\mu) \\ n_- & \text{down spins } (-\mu) \end{cases}$$

with constraint

$$n_+ + n_- = N$$

The Master Equation(s) are

$$\frac{1}{\lambda}\frac{d}{dt}n_+ = n_- e^{\beta\epsilon_-} - n_+ e^{\beta\epsilon_+}$$

$$\frac{1}{\lambda}\frac{d}{dt}n_- = n_+ e^{\beta\epsilon_+} - n_- e^{\beta\epsilon_-}$$

A measurable quantity is

$$m = n_+ - n_-$$

From the Master equations, & using $N = n_+ + n_-$,

$$\frac{1}{\lambda}\frac{d}{dt}\frac{1}{2}(N+m) = \frac{1}{2}(N-m)e^{\beta\epsilon_-} - \frac{1}{2}(N+m)e^{\beta\epsilon_+}$$

or

$$\frac{1}{\lambda}\frac{d}{dt}m = -m\left(e^{\beta\epsilon_-} + e^{\beta\epsilon_+}\right) + N\left(e^{\beta\epsilon_-} - e^{\beta\epsilon_+}\right)$$

For nuclear spins,

$$\mu \simeq 10^{-24} \text{ erg/gauss}$$

$$H \simeq 10^{4} \text{ gauss}$$

$$\Rightarrow \quad \frac{\mu H}{k_B T} \sim \frac{1}{3}\times 10^{-6} \ll 1 \quad \text{at room } T$$

$(k_B T \sim 3\times 10^{-14}$ erg$)$

Then

$$e^{\beta\epsilon_+} = e^{-\beta\mu H} \simeq 1 - \beta\mu H$$

$$e^{\beta\epsilon_-} = e^{\beta\mu H} \simeq 1 + \beta\mu H$$

so

$$\boxed{\frac{1}{\lambda}\frac{d}{dt} m \simeq -2m + 2\beta\mu H N}$$

At equilibrium, $\frac{d}{dt} m = 0$ so

$$m_\infty = N\beta\mu H$$

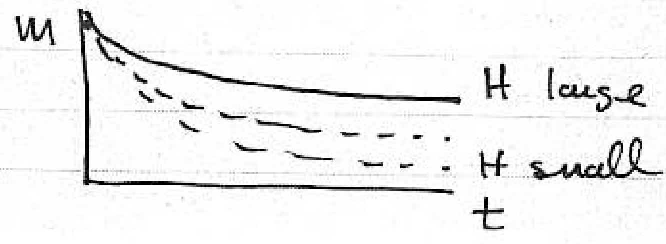
$\equiv$ spin imbalance created by H

The solution at any time is then

$$m = m(0)\, e^{-2\lambda t} + m_\infty$$

$\Rightarrow$ decay of m determined by environment & described by λ.
(motion of nucleus, for example)

$$\boxed{\text{spins}} \xrightarrow[\text{energy}]{} \boxed{\text{atomic lattice}}$$



Resonance: Can excite spins from up to down by absorption of photon:

$$\hbar\omega = \epsilon_- - \epsilon_+ = 2\mu H$$

Transition down to up by emission of $\hbar\omega$.

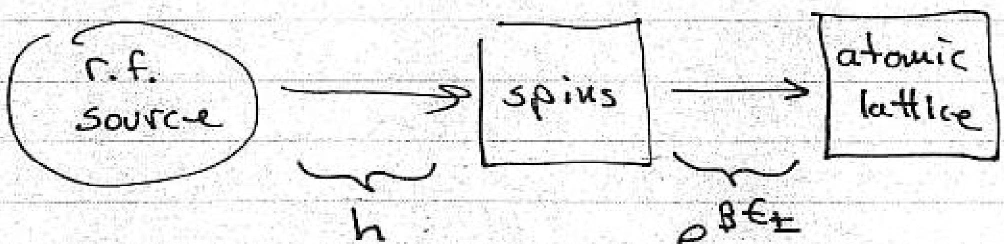
(For nuclei, $\omega \sim 10^8 \text{ sec}^{-1}$).

Drive transitions with alternating field (r.f.). Transition rate $h_{+-} = h_{-+} = h$:

$$\frac{1}{\lambda}\frac{d}{dt} n_+ = n_-\left(e^{\beta\epsilon_-} + h\right) - n_+\left(e^{\beta\epsilon_+} + h\right)$$

$$\frac{1}{\lambda}\frac{d}{dt} n_- = n_+\left(e^{\beta\epsilon_+} + h\right) - n_-\left(e^{\beta\epsilon_-} + h\right)$$

This represents energy flow



If coupling to lattice weak,

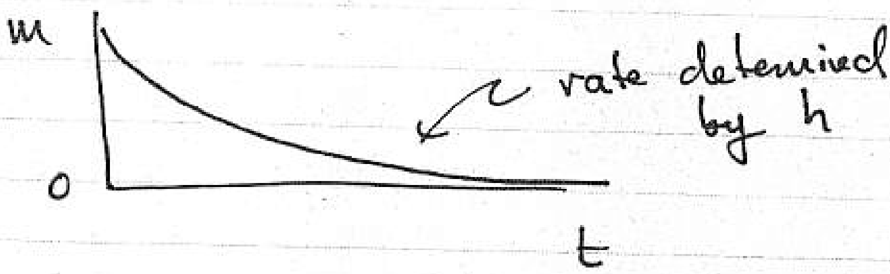
$$h \gg e^{\beta\epsilon_\pm}$$

then

$$\frac{1}{\lambda}\frac{d}{dt} m = -2h\, m$$

$$\Rightarrow \quad m = m(0)\, e^{-2ht}$$

Here, as $t \to \infty$, $m \to 0$



The reason is that if $h \gg e^{\beta\epsilon_\pm}$, the energy in $\gg$ energy out. Thus r.f. field heats spins up ($T_s \to \infty$) and all states possible.