Microscopic Mechanism for Superfluidity

From equilibrium statistical mechanics, we know that the average number of bosons in an energy state ϵ_i is

$$\langle n(\epsilon_i, T) \rangle = \frac{1}{e^{(\epsilon_i - \mu)\beta} - 1}$$

where the chemical potential μ is given in the high temperature limit by

$$\mu \simeq \frac{1}{\beta} \ln\left(\frac{N}{V} \lambda_Q^3\right)$$

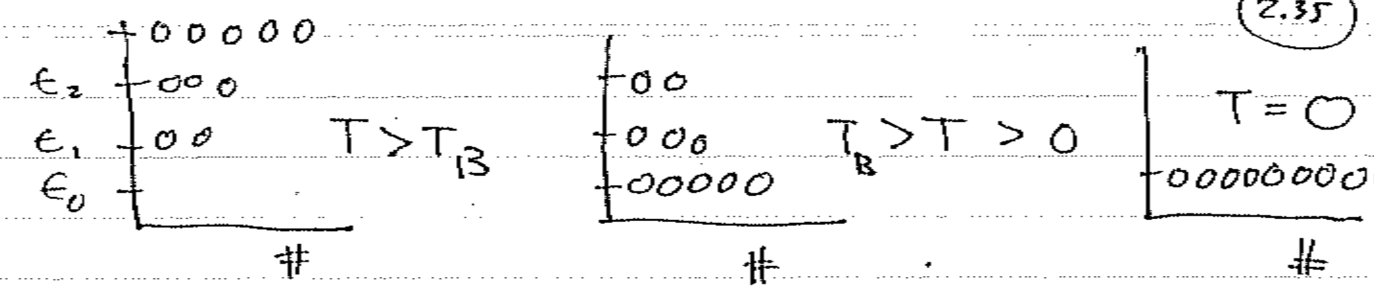
where λ_Q is quantum thermal length $\sqrt{\frac{2\pi\hbar^2}{m k_B T}}$.

In the low temperature limit, interactions between particles create a non-zero groundstate energy ϵ_0, and

$$\mu \simeq \epsilon_0 + \frac{k_B T}{N} \quad \text{for large } N \text{ as } T \to 0.$$

This means that at $T = 0$, the energy to add a particle to the system is just ϵ_0. At $T=0$, all particles are in the ground state.

A "critical" temperature T_B exists defined by equal numbers of particles in ϵ_0 and in states above ϵ_0.



This is Bose-Einstein condensation. "Condensation" here is not spatial, as in gases forming liquids. Instead it is condensation in momentum space as all particles go into the state of lowest energy kinetic.

This can provide a mechanism for superfluidity. From the point of view of viscosity, imagine a mass M moving through liquid ^4He at velocity V_M.

$$\text{(M)} \longrightarrow V_M$$

A drag can be exerted on the mass only if there is exchange of energy with the ^4He. In order for such a transfer to occur, there must be a change in V_M to V_M', and a change in energy ϵ_p to the ^4He:

$$\frac{1}{2} M V_M^2 = \frac{1}{2} M V_M'^2 + \epsilon_p .$$

Likewise, momentum will be conserved:

$$M V_M = M V_M' + p \quad \leftarrow \text{momentum change of } {}^4He$$

From momentum conservation,

$$V_M' = V_M - \frac{p}{M}$$

so $$V_M'^2 = V_M^2 + \frac{p^2}{M^2} - \frac{2}{M} V_M \cdot p$$

or $$\frac{1}{2} M V_M'^2 = \frac{1}{2} M V_M^2 + \frac{1}{2}\frac{p^2}{M} - V_M \cdot p$$

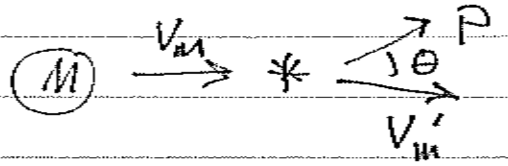
Substitution into the energy conservation equation gives

$$0 = \frac{1}{2}\frac{p^2}{M} - V_M \cdot p + \epsilon_p$$

If M is very large, this is

$$\epsilon_p \simeq p \cdot V_M = p V_M \cos\theta$$

where θ is the scattered angle from a particle (or excitation)



Solving for V_M,

$$V_M = \frac{\epsilon_p}{p\cos\theta}$$

For small θ, $(V_M)_0 \cong \frac{\epsilon_p}{p}$ which defines the minimum velocity needed to possibly collide with something in the ^4He.

Now we should be careful about assuming too much about ϵ_p. The fluid is below T_λ for ^4He, and there are some interactions. In fact, we know that excitations above the ground state are possible in the form of vortices.

We therefore leave ϵ_p unspecified and ask what the possibilities are for a <u>minimum</u> value of v_m. As a minimum,

$$\frac{d}{dp}(v_m)_0 = \frac{d}{dp}\left(\frac{\epsilon_p}{p}\right) = 0$$

But
$$\frac{d}{dp}\left(\frac{\epsilon_p}{p}\right) = \frac{1}{p}\frac{d}{dp}\epsilon_p - \frac{\epsilon_p}{p^2}$$

so
$$\frac{d}{dp}\epsilon_p = \frac{\epsilon_p}{p}$$

is the minimum condition on ϵ_p.

Let's try the case of free particles for ^4He. This would correspond to a bose condensed gas, not a fluid. Then

$$\epsilon_p = \frac{p^2}{2m}$$

so our condition is

$$\frac{p}{m} = \frac{p}{2m}$$

which is satisfied only for $p = 0$.

In this case any value of v_n is sufficient to scatter an ^4He atom and create drag.

If the ^4He interacts and creates a fluid, then excitations above the ground state include phonons as well as quantized vortex rings. Phonons have

$$\epsilon_p = c\,p$$

where c is the speed of sound, so our condition is

$$\frac{p}{m} = c \quad .$$

This defines a minimum v_n needed to create drag, and anything going slower experiences zero viscosity.

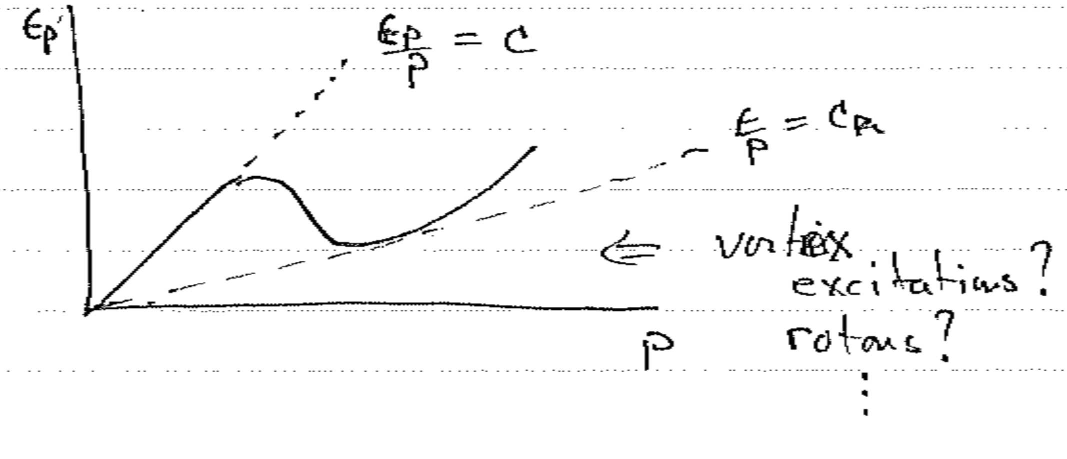
Vortices have a different energy relation:

$$\epsilon_p = c_v\, p^{1/2}$$

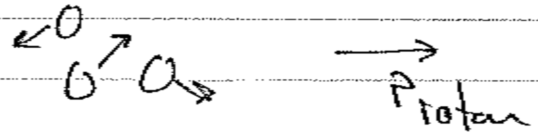
This gives the condition

$$p = (m c_v)^{2/3}$$

which again defines a minimum. Experimentally, one indeed observes multiple contributions to the excitation spectrum of ^4He:



Another type of excitation is called a "roton" and is localized like a particle, but involves correlated motions of several particles



This has a free particle dispersion ($\epsilon(p)$ relation) plus an energy gap Δ:

$$\epsilon_p = \Delta + \frac{1}{2}\frac{(p-p_0)^2}{m^*}$$

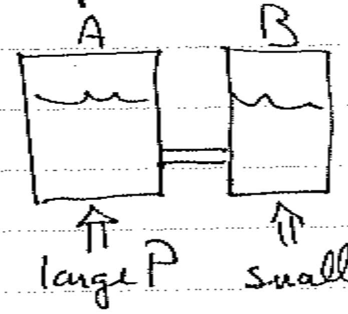
Evidence for rotons can be obtained from neutron scattering experiments, but numerical estimates and fits to data suggest that vortex excitations may be more important for determining zero viscosity than rotons.

Note: rotons are a limiting case of vortices, with diameter on the order of atomic radii.

Phenomena

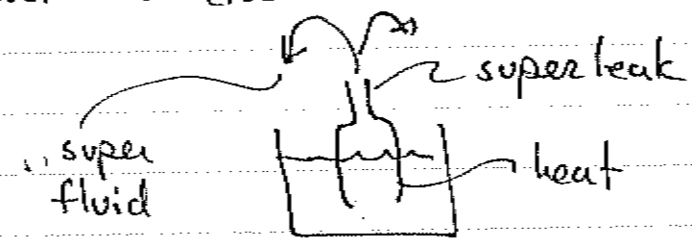
Lastly, we mention some phenomena found in super fluids. These are related to entropy, keeping in mind that a bose condensed ground state has zero entropy at $T=0$.

Mechanocaloric Effect: A "superleak" connecting two ^4He tanks is too small for normal fluid, but admits superfluid.

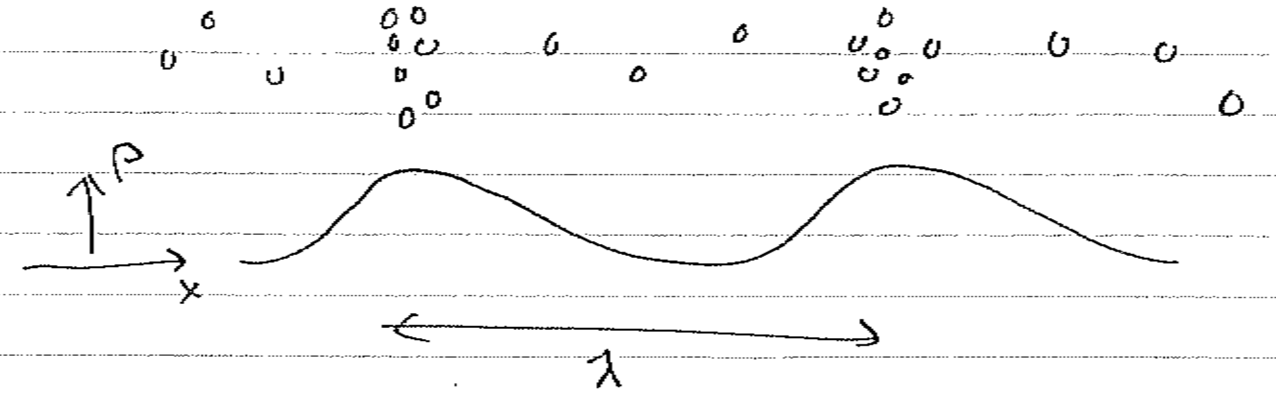


A pressure gradient is applied, driving superfluid out of A into B. This increases the entropy per mass in A, and decreases it in B. Thus temperature T in A increases, and temperature in B decreases!

Fountain Effect: This is the inverse of the mechanocaloric effect using a temperature gradient to create a fountain.



Second Sound: First sound is a density wave in which ρ_n and ρ_s oscillate
normal
in phase as a sound wave:



If ρ_s and ρ_n are 180° out of phase, then one can have cancellation such that $\rho_s + \rho_n$ is constant. There is no sound wave, but because ρ_s has zero entropy, there is an entropy wave. This appears as a travelling temperature gradient.

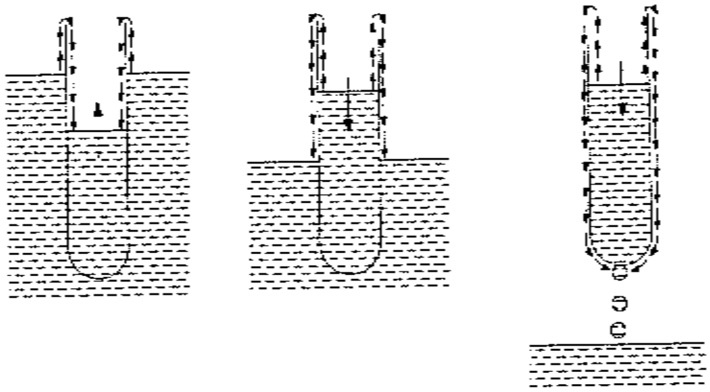


Figure 1.8 Film flow of He II over the walls of a beaker.

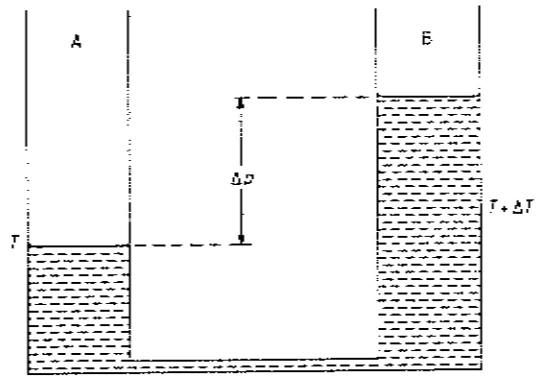
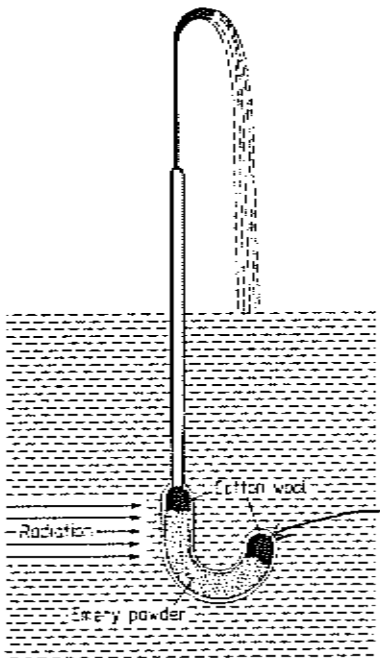


Figure 1.9 Two vessels connected by a superleak. A temperature difference between the two is accompanied by a pressure head.



open to ^4He

Figure 1.10 The helium fountain (Wilks 1967, after Allen and Jones 1938).